Coupled Inductor Design

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Summary:

Coupled inductors have many applications in power electronics. In a multiple-output buck derived regulator, filter inductor windings coupled on a common core provide vastly improved ac cross-regulation, and ripple current steering for improved filtering and reduced minimum loads.[1] Coupling the inductors in a SEPIC high power factor prereg-ulator can reduce noise by steering high frequency ripple away from the input.[2]

This paper shows how to design a coupled inductor to achieve desired objectives, using techniques discussed in previous Unitrode Seminars. Although the specific design example is the coupled inductor used in the SEPIC preregulator, the techniques use are broadly applicable.

Design Strategy:

A coupled inductor version of the SEPIC converter used in a high power factor application is described in a reference paper.[2] In that circuit, a specific *uncoupled* inductance is also required, in series with input winding of the coupled inductor. Its purpose is to steer the high frequency ripple current away from the input in order to minimize input noise filtering requirements.

In this paper, the uncoupled inductance will be obtained by integrating it into the same magnetic structure as the coupled inductance, in the form of leakage inductance between the coupled windings. The structural design will (a) provide the desired value of uncoupled inductance and (b) locate it properly in the circuit.

Normally, in high frequency magnetics design, considerable effort is expended to minimize leakage inductance and eddy current losses in the windings. Modern cores designed for switching power supplies have window areas that are long and narrow

to minimize the number of winding layers and minimize the magnetic field strength between the windings by stretching it out across a wide window. However, in this application, there is little high frequency current in the windings, so losses are not very important, and there is no desire to minimize leakage inductance but instead to utilize it for a specific purpose. So the application is better satisfied with a "low frequency" core with a more square window shape.

Application Parameters:

Coupled inductor windings $L1$, $L2$:	2 mH
Uncoupled (leakage) inductance, L_L:	0.2 mH
Total peak S.C. current, I_{pk}:	7 A
Total full load current, I_{FL}:	3 A
Total ripple current, ΔI_M:	0.5 A

Select the Proper Core Size:

Referring to the design procedure in Ref. [3], core selection is facilitated by calculating the core area product (AP) required by the application, and relating this calculation to the APs of available cores. This step is not necessary, it simple reduces the number of trial solutions that would be required to find the optimum core. (The core area product is the magnetic cross section area times the window area. Some manufacturers call this WaAc, others AwAe.)

At low frequencies, core size is dictated by I^2R losses in the windings and the saturation flux density of the core material. At high frequencies, core losses dominate -- the core cannot be operated anywhere near saturation flux density. Different AP calculations are required for these two cases. Since the high frequency ripple current is small in this coupled inductor application, the saturation limited

case applies. The saturation limited formula (SI units) is:

$$AP = \left(\frac{L\,I_{PK}\,I_{FL} \cdot 10^4}{420\,K\,B_{max}} \right)^{1.31} \quad cm^4$$

where B_{max} for ferrite core material is taken to be 0.3 Tesla (3000 Gauss), and K is the core window utilization factor, taken to be 0.5 in this case with two windings. Using the values of this application:

$$AP = \left(\frac{.002 \cdot 7 \cdot 3 \cdot 10^4}{420 \cdot 0.5 \cdot 0.3} \right)^{1.31} = 12.0 \ cm^4$$

(The core loss limited formula results in an AP of 0.29 cm^4, indicating its irrelevance in this case.)

An AP of 12 is a fairly large core. An EC70 core with an AP of 17.8 could satisfy this requirement, but a PQ50/50 with an AP of 14.2 was chosen because it is smaller and more compact, it is better shielded, and it has a more square window area which will help achieve the desired leakage inductance. (The smaller PQ40/40 core with AP of 5.0 was tried, but proved much too small.)

PQ50/50 CORE PARAMETERS:

Magnetic cross-section area, *Ae:*	3.28 cm^2
Winding window area, *Aw:*	4.33 cm^2
Window width, *Ww:*	3.6 cm
Winding width (bobbin), *bw:*	3.2 cm
Winding height (bobbin), *hw:*	1 cm
Mean length per turn, *MLT:*	10 cm
Free air thermal resistance, *Rr:*	8 °C/W

The next step is to calculate the minimum number of turns that will take the core to B_{max} at the peak current limit. (Fewer turns will require a smaller gap to achieve the required inductance and will take the flux beyond B_{max}.)

$$N_{MIN} = \frac{L\,I_{PK} \cdot 10^4}{B_{max}\,A_e} = \frac{.002 \cdot 7 \cdot 10^4}{0.3 \cdot 3.28} = 142 \ turns$$

Using the minimum number of turns, next calculate the center-pole gap length required to achieve the desired inductance $L1$ (same as $L2$). Compared to the gap, the energy stored in the

ferrite is insignificant. The gap alone determines the energy stored, and therefore the inductance value. Relative permeability, μ_r, equals 1 in the gap:

$$\ell_{gap} = \frac{\mu_o\,\mu_r\,N^2\,A_e}{L} \cdot 10^{-2} \quad cm$$

$$\ell_{gap} = \frac{4\pi \cdot 10^{-7} \cdot 142^2 \cdot 3.28}{.002} \cdot 10^{-2} = .415 \ cm$$

The gap must be placed entirely in the center-leg. The outer legs should be in intimate contact. *Do not shim the core halves apart. Although the desired inductance may be obtained this way, the leakage inductance will not be in the desired circuit location.*

The permissible power loss in the windings for a 32°C temperature rise is:

$$Pw = \Delta T / R_T = 32/8 = 4 \ Watts$$

At full power (200W), the rms current in winding $L2$ is 200W/200V = 1 A rms. In winding $L1$, worst case rms current is at low line: 200W/100V = 2 A rms. Since the turns are equal, we can think of this as 3 A rms through a single winding. The maximum allowable resistance of this composite winding is:

$$R_{Wmax} = Pw / I_{FL}^2 = 4/3^2 = .444 \ \Omega$$

The resistance per centimeter wire length is:

$$Rw/cm = Rw/(N \cdot MLT)$$
$$= .444/(142 \cdot 10) = .313 \ m\Omega/cm$$

For simplicity, we will use the same wire size for both windings (most of the time the current in $L1$ is considerably less than 2 A). Thus each wire will have half the copper area and twice the resistance: 2x.313 = .616 mΩ/cm. Looking in the wire tables, AWG22 (.64mm) has a resistance of .530 mΩ/cm at 20°C, or .708 mΩ/cm at 100°C.

Referring to the winding structure in Figure 1, the two windings will be wound separately. The first winding, closest to the gapped center-leg, will be winding $L2$. The leakage inductance energy is stored in the non-magnetic region within and between the two windings. This leakage inductance flux and its associated energy is linked to the outer
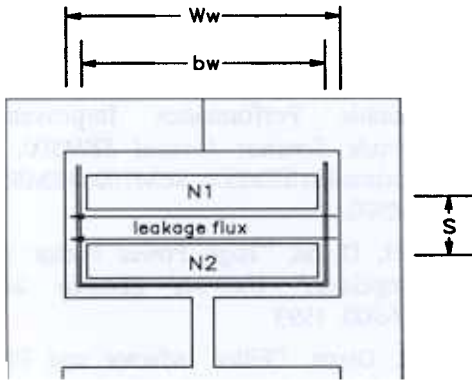
Fig 1. - Winding Structure

winding, $L1$, (assuming the outer legs are not gapped). The amount of leakage inductance is determined by the spacing between the windings.

First the structure of the two windings is established. AWG22 insulated diameter is .71mm. The bobbin width is 32mm, allowing 32/0.71 = 46 turns per layer. 142 turns will require 3.1 layers. Why not go to 4 full layers with larger wire to reduce losses? Working backwards, 4 full layers has 142/4 or 36 turns per layer. 32mm/36t = .89mm wire diameter. According to the wire table, AWG20 has an insulated diameter of .89mm and a resistance of only .445 mΩ/cm at 100ωC. The height of each 4-layer winding is 4·.89mm = 3.56mm, or .356cm. The total height available for both windings is 1 cm., so there is plenty of room.

Establish the Leakage Inductance Value:

Figure 1 shows that the flux representing the leakage inductance energy extends across the entire core window. Even though these flux lines proceed through the outer core leg to form closed loops, the energy stored within the core material is negligible. (Ferrite permeability is so high the field intensity is negligible within the core.) So the length of the *effective* leakage inductance field is the width of core window, Ww. The cross-sectional area of the leakage inductance field is the cylindrical area between the windings, which equals the mean length of a turn, MLT, multiplied by the separation, S, between the windings. The field actually extends

into each winding, tailing off from the center to the outer edges next to the core. As an approximation, the effective separation between the windings includes 1/3 of the height of each winding, as shown in Fig.1. The entire region where the leakage inductance energy is stored is non-magnetic, with μ_r=1. The same inductance formula used before can now be used to calculate the leakage inductance value. Only the terms representing the area and length of the field have been changed:

$$L_L = \frac{\mu_o \mu_r N^2 (MLT \cdot S)}{W_W} \cdot 10^{-2} \quad cm$$

$$L_L = \frac{4\pi \cdot 10^{-7} \cdot 142^2 \cdot (10 \cdot S)}{3.6} \cdot 10^{-2} = 0.7 S \text{ mH}$$

Since a leakage inductance of 0.2mH is desired:

$$S = L_L/0.7 = 0.2/0.7 = .286 \text{ cm}$$

To obtain the actual spacing between the windings, subtract 1/3 of the winding height (twice).

$$\text{Spacing} = .286 - 2(.284/3) = .097 \text{ cm}.$$

Using a PQ core, the actual leakage inductance will be perhaps 25% smaller than calculated above. This can be corrected by a corresponding increase in the S value. The reason for the reduced inductance is that a considerable portion of the windings lie outside the core window, where the leakage inductance field is not bounded by the window width Ww. These flux lines must bend around to reach the core, and the increased length of the field reduces the field strength and energy in this region. A pot core will give a result much closer to the calculated value.

Reluctance Modeling:

A previous Seminar topic[4] showed how to use reluctance modeling and duality to translate the physical structure of a magnetic device into its equivalent inductance values and their circuit locations. This technique is extremely valuable for understanding and optimizing the structure of any magnetic device.

Figure 2 shows the structure of this coupled inductor with the various reluctances indicated. Reluctance of the core legs is not shown—it is so
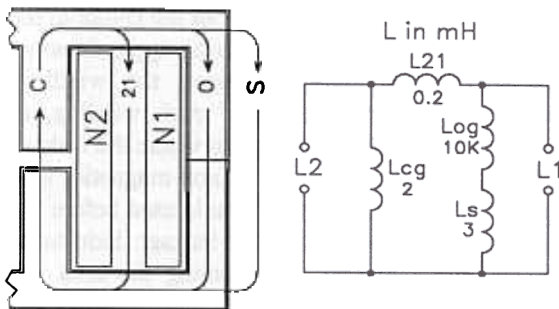
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Fig 2. - Modeling the Coupled Inductor

small it is negligible. Duality converts the reluctances to permeances, with parallel reluctances becoming series permeances, etc. Permeance multiplied by N^2 becomes inductance.

Note on the left that the reluctance of the stray field is in parallel with the reluctance of the outer leg. The stray field reluctance is *not* negligible. It is in fact comparable to the reluctance of the center-leg gap which is the determinant of the coupled inductance value. However, with the outer legs tightly mated to minimize the outer-leg gap, the reluctance of the outer leg is extremely low. This shunts the stray field reluctance, rendering it unimportant. In the equivalent electrical circuit shown on the right, shunt reluctances are converted into series inductances, with low reluctance becoming high inductance. Note how the inductance component derived from the outer leg is now an extremely large value in series with an otherwise significant inductance contribution from the stray field. Effectively, L_{og} and L_s could be erased from the circuit diagram. Not only is the stray field voided, the leakage inductance $L21$ appears in series with $L2$, as desired, not $L1$. (An ideal transformer could be added at either $L1$ or $L2$ terminals to simulate the isolation between the actual windings.)

So putting a gap in the outer leg will raise its reluctance, and lower its inductance in the equivalent circuit. This not only makes the stray field a part of the coupled inductance, but the leakage inductance is no longer exclusively associated with $L1$, spoiling the ripple current steering intended in the application.

References:

[1] L. H. Dixon, "Coupled Filter Inductors in Multiple Output Buck Regulators Provide Dramatic Performance Improvement," *Unitrode Seminar Manual SEM500*, 1986 (Reprinted in SEM600, SEM700, SEM800 and SEM900.)

[2] L. H. Dixon, "High Power Factor SEPIC Preregulator," *Unitrode Seminar Manual SEM-900*, 1993

[3] L.H. Dixon, "Filter Inductor and Flyback Transformer Design for Switching Power Supplies," *Unitrode Seminar Manual SEM400*, 1985 (Reprinted in SEM600, SEM700, SEM800, and SEM900)

[4] L. H. Dixon, "How to Place Leakage and Wiring Inductances in the High Frequency Model," *Unitrode Seminar Manual SEM600*, 1988 (Reprinted in SEM700, SEM800, and SEM900.)

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