On the Definition of Total Harmonic Distortion and its Effect on Measurement Interpretation

Doron Shmilovitz

Abstract— **The existence of two different definitions for total harmonic distortion (one in comparison to the fundamental and one in comparison to the signals rms) might cause ambiguity and misinterpretation of measured data. The difference between those definitions is stressed out in this letter. It is suggested that THD measurements in the context of power systems should always adopt the first definition and never the second.**

***Index Terms*-- Harmonic distortion, nonlinear load, nonsinusoidal waveforms, power measurements, power meters.**

I. Introduction

Total harmonic distortion (THD) is an important figure of merit, used to quantify the level of harmonics in voltage or current waveforms. Two different definitions for THD may be found in the literature. According to one definition, the harmonic content of a waveform is compared to its fundamental, [1],[2]. By the second definition, the harmonic content of a waveform is compared to the waveforms rms value, [3]. In order to distinguish between the two, the former is occasionally denoted by THD_F and the second by THD_R. For instance, current THDs are defined as:

$$THD_F = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^{\,2}}}{I_1} \quad ; \quad THD_R = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^{\,2}}}{\sqrt{\sum_{n=1}^{\infty} I_n^{\,2}}} \tag{1}$$

where I_n are either the rms values or the amplitudes of the harmonics. At low values of THD there is no much difference between the two. However, the two definitions may cause ambiguity, confusion, and misinterpretation when measuring waveforms of high harmonic content.

The relation and difference between the two definitions of THD are stressed in this letter. Usage of THD_F rather than THD_R is advocated.

II. The Relation Between THD_F and THD_R

Actually there is a consensus as to the basic definition of THD in the context of power measurements, by which it is defined with respect to the fundamental, [1],[2],[4].

It seems that the second definition, THD_R was inherited from the area of audio amplifiers, where the THD serves as a measure of the systems linearity and it's numerical value is always much less than 1 (practically it ranges from 0.1% - 0.3% in Hi-Fi systems up to a few percent in conventional audio systems). Thus, for this range of THD values, the error caused by mixing up the two definitions of THD was acceptable. For instance, if the actual THD (THD_F) is 10%, THD_R will have the value of 9.95% (less than 0.5% difference). Moreover, with the older type, analog distortion analyzers incorporated for amplifiers testing, it is easier to measure THD_R; the nominator in (1) is obtained by filtering out the fundamental with a notch filter, and the denominator is simply the signals rms value. These two quantities are related by (2) and plotted in Fig.1.

$$THD_R = \frac{THD_F}{\sqrt{1+THD_F^2}} \tag{2}$$

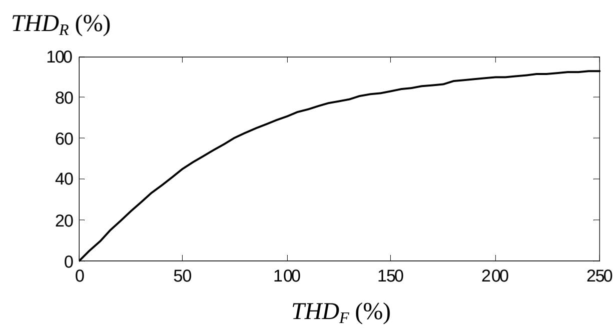


Fig. 1. Total harmonic distortion in percent of the signals rms versus its basic definition (in percent of the fundamental).

Evidently, at high values of THD the difference becomes essential. THD_R cannot exceed 100% whereas THD_F may reach higher values when the spectral energy of the harmonics exceeds that of the fundamental (mathematically, it may reach infinity if a waveform contains no fundamental).

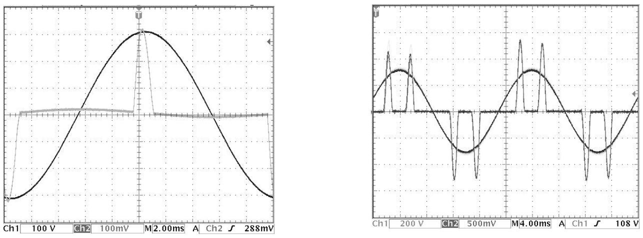
III. Interpretation and Accuracy Issues

High current THDs are quite common in electronic loads, [5],[6]. For instance, values of 140%-170% are typical for currents drawn by peak detection rectifiers, see Fig. 2. Fig. 1 shows that at high THDs, large variations in THD_F manifest in very little variation in THD_R, and vise versa, small differences of THD_R are in fact large differences in the THD (THD_F) content. This can be quantified by the sensitivity of the THD with respect to variations in THD_R, defined in by:

$$S_{THD_R}^{THD_F} = \frac{\partial THD_F}{\partial THD_R} \cdot \left(\frac{THD_R}{THD_F} \right) \tag{3}$$

This work was supported in part by the Israeli ministry of national infrastructure and energy, under grant No.551-171.

Doron Shmilovitz is with the Faculty of Engineering, Tel Aviv University, Tel Aviv, Israel (e-mail: shmilo@eng.tau.ac.il).



(a) (b)

Fig. 2. Peak detection rectifiers typical wave forms. (a) single phase (94Watt, pc load), 100V/div, 1A/div, THD≅178% (b) three phase, low load (880Watt, ASD), 200V/div, 5A/div, THD≅172%.

which yields:

$$S_{THD_R}^{THD_F} = \frac{\sqrt{1+THD_F^{\,2}}}{\sqrt{1+THD_F^{\,2}} - THD_F} \tag{4}$$

This sensitivity is quite high at high THDs, for instance, for THD_F equal to 170%, the sensitivity is above 7, inaccuracy may be caused, in two ways:

A. Human perception

Values of 87% or 89% may be thought to be pretty much the same. However, should the above numbers represent THD_R, this actually represents a difference of 19%, between 176% and 195%, respectively in terms of THD_F. Thus, what seems to be a 2% difference (in terms of THD_R) is, in fact, a 10% difference in THD_F. A 10% difference will be caused in the distortion factor, DF as well (DF(THD_F=176%)=45% and DF(THD_F=195%)=49%), where DF is defined as:

$$DF = \frac{1}{\sqrt{1+THD_F^{\,2}}} = \sqrt{1-THD_R^{\,2}} \tag{5}$$

B. Instrumentation accuracy

The effect of instrumentation finite accuracy is similar to the one above. The specified THD measurement accuracies of four power quality analyzer models are listed in Table I. As can be seen, all the accuracies contain terms, which are independent of the reading. That implies reduced accuracy if THD_R is used.

TABLE I
SPECIFIED ACCURACIES OF THD MEASUREMENT FOR DIFFERENT POWER ANALYZER MODELS

POWER QUALITY ANALYZER MODEL	THD MEASUREMRNT ACCURECY
FLUKE 43B	3%+8COUNTS
FLUKE 41B	0.03*READING+2%
LEM LH1060	3%
AGILENT 6813B	0.05*READING+0.1%

IV. EXPERIMENTAL RESULTS

The power quality analyzers listed in Table I have been compared when measuring highly distorted signals. The measurement accuracy was always higher when THD_F was used. Measurements results for the single-phase rectifier (Fig. 2(a)) are summarized in Table II. The average measured values are referred to as accurate ones. The measurement error, ΔTHD, is calculated as the difference of the measured value from the average, in percent of the average value.

$$\Delta THD_R(\%) = \frac{\left|THD_R - \langle THD_R \rangle\right|}{\langle THD_R \rangle}\cdot 100\%$$
$$\Delta THD_F(\%) = \frac{\left|THD_F - \langle THD_F \rangle\right|}{\langle THD_F \rangle}\cdot 100\% \tag{6}$$

In this test case, the average THD_R (<THD_R>) was 87.2% and the average THD_F (<THD_F>) was 178% (which doesn't comply with (2)). Not only the measurement errors are higher when using THD_R, but also the error generated when computing THD_F from THD_R is enormous. For instance, THD_R=93.6% (Table II) yields THD_F=266%, an error of 49%!

TABLE II
EXAMPLE OF MEASUREMENT ERROR

	THD_F	ΔTHD_F(%)	THD_R	ΔTHD_R(%)
FLUKE 43B	183%	2.73%	84%	3.7%
FLUKE 41B	173%	2.73	82%	6%
LEM LH1060	181%	1.69%	89.2%	2.3%
Agilent 6813B	175%	1.70%	93.6%	7.3%

V. CONCLUSIONS

It has been shown that THD_F is a much better measure of harmonics content. Employment of THD_R in measurements may yield high errors in significant quantities such as power-factor and distortion-factor, derived from THD measurement, [7]. Modern power analyzers incorporate DFT based algorithms (as opposed to older, analog analyzers). Thus there is no reason to include THD_R, even as an optional measurement, as it may cause errors and misinterpretation.

VI. References

[1] IEC, "Electromagnetic compatibility (EMC)- Part 4, section 7: General guide on harmonics and interharmonics measurements and instrumentation, for power supply system and equipment and equipment connected thereto," 1991.

[2] A.E. Emanuel, "Power in Nonsinusoidal Situations – A Review of Definitions and physical Meaning," *IEEE Trans. On Power Delivery*, Vol. 5, No. 3, July 1990, pp. 1377-1389.

[3] IEC, "International Electrotechnical Vocabulary- chapter 131: Electric and magnetic circuits," 1978.

[4] IEEE Working group in Nonsinusoidal Situations, "Practical Definitions for Power Systems with Nonsinusoidal Waveforms and Unbalanced Loads: A Discussion," *IEEE Trans. On Power Delivery*, vol. 11, no.1, Jan. 1996, pp. 79-87.

[5] T. A. Buchh, A. Jr. Domijan, "Harmonic effect of electric vehicle loads," *IASTED, International Journal of Power & Energy Systems*, vol. 21, no. 2, 2001, pp. 62-6.

[6] D. Czarkowski, A. Jr. Domijan, "Harmonic content of PWM adjustable speed drive waveforms-analysis and metering implications," in *Proceedings of the ICHQP, 7th International Conference on Harmonics and Quality of Power*, 1996, pp. 48-53.

[7] J. Arrillaga, N. R. Watston, S. Chaen, *Power Systems Quality Assessment*, New York: Wiley, 2000.