

# ***Field Orientated Control of 3-Phase AC-Motors***

Literature Number: BPRA073  
Texas Instruments Europe  
February 1998

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# ***Field Orientated Control of 3-Phase AC-Motors***

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## **ABSTRACT**

The principle of vector control of electrical drives is based on the control of both the magnitude and the phase of each phase current and voltage. For as long as this type of control considers the three phase system as three independent systems the control will remain analog and thus present several drawbacks. Since high computational power silicon devices, such as the TMS320F240 from TI, came to market it has been possible to realize far more precise digital vector control algorithms. The most common of these accurate vector controls is presented in this document: the Field Orientated Control, a digital implementation which demonstrates the capability of performing direct torque control, of handling system limitations and of achieving higher power conversion efficiency.

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## **1. Introduction**

During the last few years the field of controlled electrical drives has undergone rapid expansion due mainly to the advantages of semiconductors in both power and signal electronics and culminating in micro-electronic microprocessors and DSPs. These technological improvements have enabled the development of really effective AC drive control with ever lower power dissipation hardware and ever more accurate control structures. The electrical drive controls become more accurate in the sense that not only are the DC current and voltage controlled but also the three phase currents and voltages are managed by so-called *vector controls*. This document describes the most efficient form of vector control scheme: the Field Orientated Control. It is based on three major points: the machine current and voltage space vectors, the transformation of a three phase speed and time dependent system into a two co-ordinate time invariant system and effective Pulse Width Modulation pattern generation. Thanks to these factors, the control of AC machine acquires every advantage of DC machine control and frees itself from the mechanical commutation drawbacks. Furthermore, this control structure, by achieving a very accurate steady state and transient control, leads to high dynamic performance in terms of response times and power conversion. These different aspects are discussed in the following chapters.

## **2. Classic AC drives**

AC motor control structures generally apply three 120° spatially displaced sinusoidal voltages to the three stator phases. In most of the classic AC drives the generation of the three sine waves is based on motor electromechanical characteristics and on an equivalent model for the motor in its steady state. Furthermore, the control looks like three separate single phase system controls rather than one control of a three phase system. Some major common drawbacks are presented in this chapter [1]:

- 
- The machine models and characteristics used are valid only in steady state. This causes the control to allow high peak voltage and current transients. These damage not only the drive dynamic performance but also the power conversion efficiency. Additionally, the power components must be oversized to withstand the transient electrical spikes.
  - Great difficulty in controlling the variables with sinusoidal references: PI regulators can not perform a sinusoidal regulation without damaging the sinusoidal reference, and hysteresis controllers introduce high bandwidth noise into the system that is hard to filter out.
  - No three phase system imbalance management. No consideration of the phase interactions.
  - Finally, the control structure must be dedicated according to motor type (asynchronous or synchronous).

The following chapters present the Field Orientated Control of AC drives. This control solution overcomes each of these drawbacks and thus improves the overall effectiveness of the AC drive. Detailed explanations and references to other helpful documents gives the reader a good understanding of the control structure and of the immediate benefits of such a solution.

### 3. Field Orientated Control

The Field Orientated Control (FOC) [1][3] consists of controlling the stator currents represented by a vector. This control is based on projections which transform a three-phase time and speed dependent system into a two co-ordinate ( $d$  and  $q$  co-ordinates) time invariant system. These projections lead to a structure similar to that of a DC machine control. Field orientated controlled machines need two constants as input references: the torque component (aligned with the  $q$  co-ordinate) and the flux component (aligned with  $d$  co-ordinate). As Field Orientated Control is simply based on projections the control structure handles instantaneous electrical quantities. This makes the control accurate in every working operation (steady state and transient) and independent of the limited bandwidth mathematical model. The FOC thus solves the classic scheme problems, in the following ways:

- the ease of reaching constant reference (torque component and flux component of the stator current)
- the ease of applying direct torque control because in the  $(d,q)$  reference frame the expression of the torque is:

$$m \propto \psi_R i_{sq}$$

By maintaining the amplitude of the rotor flux ( $\psi_R$ ) at a fixed value we have a linear relationship between torque and torque component ( $i_{sq}$ ). We can then control the torque by controlling the torque component of stator current vector.

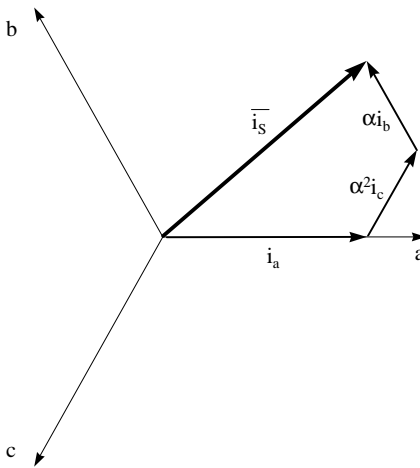
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### 3.1 Space Vector definition and projection

The three-phase voltages, currents and fluxes of AC-motors can be analyzed in terms of complex space vectors [1][6]. With regard to the currents, the space vector can be defined as follows. Assuming that  $i_a$ ,  $i_b$ ,  $i_c$  are the instantaneous currents in the stator phases, then the complex stator current vector  $\bar{i}_s$  is defined by:

$$\bar{i}_s = i_a + \alpha i_b + \alpha^2 i_c$$

where  $\alpha = e^{j\frac{2}{3}\pi}$  and  $\alpha^2 = e^{j\frac{4}{3}\pi}$ , represent the spatial operators. The following diagram shows the stator current complex space vector:



**Figure 1: Stator current space vector and its component in (a,b,c)**

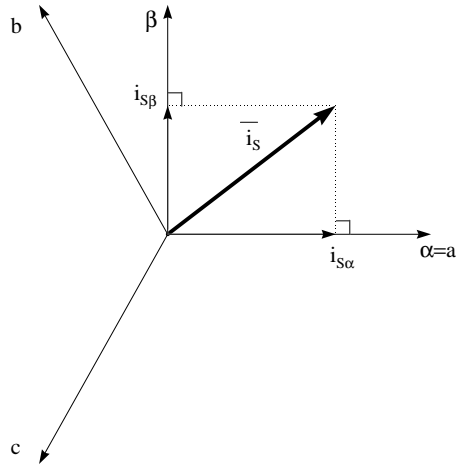
where (a,b,c) are the three phase system axes. This current space vector depicts the three phase sinusoidal system. It still needs to be transformed into a two time invariant co-ordinate system. This transformation can be split into two steps:

- (a,b,c) $\Rightarrow$ ( $\alpha,\beta$ ) (the Clarke transformation) which outputs a two co-ordinate time variant system
- ( $\alpha,\beta$ ) $\Rightarrow$ ( $d,q$ ) (the Park transformation) which outputs a two co-ordinate time invariant system

This is explained in the following chapter.

#### 3.1.1 The (a,b,c)- $\rightarrow$ ( $\alpha,\beta$ ) projection (Clarke transformation)

The space vector can be reported in another reference frame with only two orthogonal axis called ( $\alpha,\beta$ ). Assuming that the axis  $a$  and the axis  $\alpha$  are in the same direction we have the following vector diagram:



**Figure 2: Stator current space vector and its components in (a,b)**

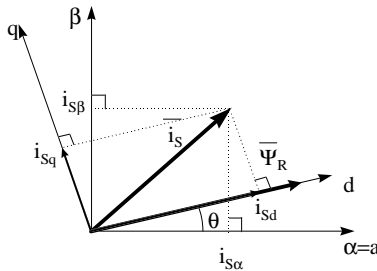
The projection that modifies the three phase system into the  $(\alpha, \beta)$  two dimension orthogonal system is presented below.

$$\begin{cases} i_{s\alpha} = i_a \\ i_{s\beta} = \frac{1}{\sqrt{3}}i_a + \frac{2}{\sqrt{3}}i_b \end{cases}$$

for a TMS320F240 software implementation refer to report (BPRA048). We obtain a two co-ordinate system  $\begin{pmatrix} i_{s\alpha} \\ i_{s\beta} \end{pmatrix}$  that still depends on time and speed.

### 3.1.2 The $(\alpha, \beta) \rightarrow (d, q)$ projection (Park transformation)

This is the most important transformation in the FOC. In fact, this projection modifies a two phase orthogonal system  $(\alpha, \beta)$  in the  $d, q$  rotating reference frame. If we consider the  $d$  axis aligned with the rotor flux, the next diagram shows, for the current vector, the relationship from the two reference frame:



**Figure 3: Stator current space vector and its component in (a,b) and in the d,q rotating reference frame**



---

where  $\theta$  is the rotor flux position. The flux and torque components of the current vector are determined by the following equations:

$$\begin{cases} i_{sd} = i_{s\alpha} \cos\theta + i_{s\beta} \sin\theta \\ i_{sq} = -i_{s\alpha} \sin\theta + i_{s\beta} \cos\theta \end{cases}$$

These components depend on the current vector ( $\alpha, \beta$ ) components and on the rotor flux position; if we know the right rotor flux position then, by this projection, the  $d, q$  component becomes a constant. For TMS320F240 software implementation refer to report

(BPRA048). We obtain a two co-ordinate system  $\begin{pmatrix} i_{sd} \\ i_{sq} \end{pmatrix}$  with the following characteristics:

- two co-ordinate time invariant system
- with  $i_{sd}$  (flux component) and  $i_{sq}$  (torque component) the direct torque control is possible and easy.

### 3.1.3 The $(d,q) \rightarrow (\alpha,\beta)$ projection (inverse Park transformation)

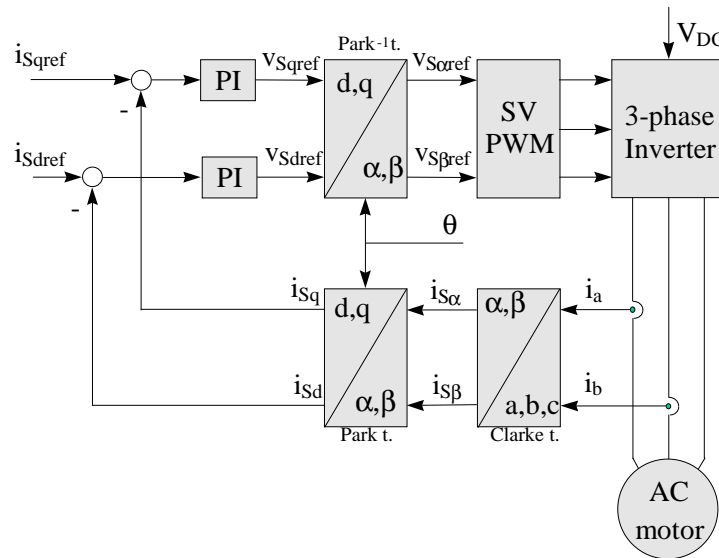
Here, we introduce from this voltage transformation only the equation that modifies the voltages in  $d, q$  rotating reference frame in a two phase orthogonal system:

$$\begin{cases} v_{s\alpha ref} = v_{sdref} \cos\theta - v_{sqref} \sin\theta \\ v_{s\beta ref} = v_{sdref} \sin\theta + v_{sqref} \cos\theta \end{cases}$$

The outputs of this block are the components of the reference vector that we call  $\bar{V}_r$ ;  $\bar{V}_r$  is the voltage space vector to be applied to the motor phases. For TMS320F240 software implementation refer to report (BPRA048).

## 3.2 The basic scheme for the FOC

The following diagram summarizes the basic scheme of torque control with FOC [1][2][3]:



**Figure 4: Basic scheme of FOC for AC-motor**

Two motor phase currents are measured. These measurements feed the Clarke transformation module. The outputs of this projection are designated  $i_{s\alpha}$  and  $i_{s\beta}$ . These two components of the current are the inputs of the Park transformation that gives the current in the  $d,q$  rotating reference frame. The  $i_{sd}$  and  $i_{sq}$  components are compared to the references  $i_{sdref}$  (the flux reference) and  $i_{sqref}$  (the torque reference). At this point, this control structure shows an interesting advantage: it can be used to control either synchronous or induction machines by simply changing the flux reference and obtaining rotor flux position. As in synchronous permanent magnet motors, the rotor flux is fixed (determined by the magnets) there is no need to create one. Hence, when controlling a PMSM,  $i_{sdref}$  should be set to zero. As induction motors need a rotor flux creation in order to operate, the flux reference must not be zero. This conveniently solves one of the major drawbacks of the “classic” control structures: the portability from asynchronous to synchronous drives. The torque command  $i_{sqref}$  could be the output of the speed regulator when we use a speed FOC. The outputs of the current regulators are  $v_{sdref}$  and  $v_{sqref}$ ; they are applied to the inverse Park transformation. The outputs of this projection are  $v_{s\alpha ref}$  and  $v_{s\beta ref}$  which are the components of the stator vector voltage in the  $\alpha,\beta$  stationary orthogonal reference frame. These are the inputs of the Space Vector PWM. The outputs of this block are the signals that drive the inverter. Note that both Park and inverse Park transformations need the rotor flux position. Obtaining this rotor flux position depends on the AC machine type (synchronous or asynchronous machine). Rotor flux position considerations are made in a following paragraph.

### 3.3 The input for the FOC

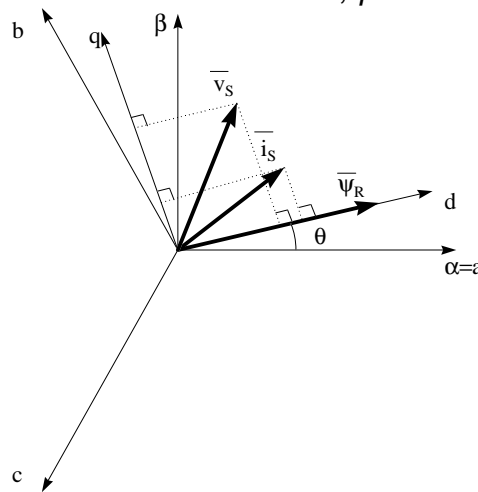
Fundamental requirements for the FOC are a knowledge of two phase currents (as the motor is star-connected, the third phase current is also known, since  $i_a + i_b + i_c = 0$ ), and the rotor flux position.

#### 3.3.1 Current sampling

The measured phase currents  $i_a$  and  $i_b$  are sampled and converted by an A/D converter. The correct working of the FOC depends on the true measurement of these currents.

#### 3.3.2 Rotor flux position

**Knowledge of the rotor flux position is the core of the FOC.** In fact if there is an error in this variable the rotor flux is not aligned with d-axis and  $i_{sd}$  and  $i_{sq}$  are incorrect flux and torque components of the stator current. The following diagram shows the (a,b,c), ( $\alpha,\beta$ ) and (d,q) reference frames, and the correct position of the rotor flux, the stator current and stator voltage space vector that rotates with d,q reference at synchronous speed.



**Figure 5: Current, voltage and rotor flux space vectors in the d,q rotating reference frame and their relationship with a,b,c and a,b stationary reference frame**

The measure of the rotor flux position is different if we consider synchronous or induction motor.

- In the synchronous machine the rotor speed is equal to the rotor flux speed. Then  $\theta$  (rotor flux position) is directly measured by position sensor or by integration of rotor speed [1].
- In the induction machine the rotor speed is not equal to the rotor flux speed (there is a slip speed), then it needs a particular method to calculate  $\theta$ . The basic method is the use of the current model [1][2][3] which needs two equations of the motor model in d,q reference frame.

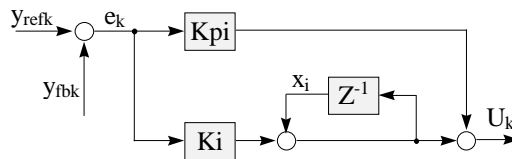
### 3.4 Conclusion

Thanks to FOC it becomes possible to control, directly and separately, the torque and flux of AC machines. Field Orientated Controlled AC machines thus obtain every DC machine advantage: instantaneous control of the separate quantities allowing accurate transient and steady state management. In addition to this advantage, Field Orientated Controlled AC machines solve the mechanical commutation problems inherent with DC machines. TMS320F240, by providing high CPU power and highly versatile motor control dedicated peripherals, makes the use of DC machines obsolete in terms of power conversion efficiency and system reliability, when compared with FOC AC machines.

## 4. The PI regulator

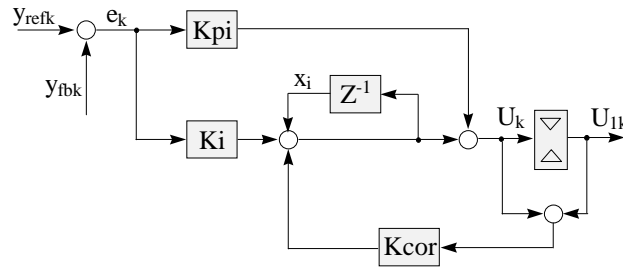
An electrical drive based on the Field Orientated Control needs two constants as control parameters: the torque component reference  $I_{sqref}$  and the flux component reference  $I_{sdref}$ . The classic numerical PI (Proportional and Integral) regulator is well suited to regulating the torque and flux feedback to the desired values as it is able to reach constant references, by correctly setting both the P term ( $K_{pi}$ ) and the I term ( $K_i$ ) which are respectively responsible for the error sensibility and for the steady state error. The numerical expression of the PI regulator is as follows:

$$U_k = K_{pi}e_k + K_i e_k + \sum_{n=0}^{k-1} e_n \text{ which can be represented by the following figure:}$$



**Figure 6: Classical Numerical PI Regulator Structure**

According to [4], the limiting point is that during normal operation, or during the tests, large reference value variations or large disturbances may occur, resulting in saturation and overflow of the regulator variables and output. If they are not controlled, this kind of non-linearity damages the dynamic performance of the system. To solve this problem, one solution is to add to the previous structure a correction of the integral component as depicted in the following diagram:



**Figure 7: Numerical PI Regulator with Correction of the Integral Term**

The integral term correction algorithm in a high level language is given below:

```

INPUT     $y_{refk}, y_{fbk}$ 
           $e_k = y_{refk} - y_{fbk}$ 
           $u_k = x_i + K_{pi} e_k$ 
           $u_{lk} = u_k$ 
          IF  $u_k > u_{max}$  THEN  $u_{lk} = u_{max}$ 
          IF  $u_k < u_{min}$  THEN  $u_{lk} = u_{min}$ 
OUTPUT    $u_{lk}$ 
           $e_{lk} = u_k - u_{lk}$ 
           $x_i = x_i + K_i e_k + K_{cor} e_{lk}$ 

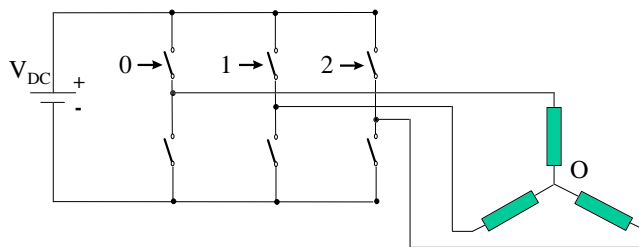
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With  $u_{max}$ ,  $u_{min}$  we mean the limitations of the output variable.

## 5. The Space Vector PWM

### 5.1 The 3-phase Inverter

The structure of a typical 3-phase power inverter is shown in Figure 8, where  $V_A$ ,  $V_B$ ,  $V_C$  are the voltages applied to the star-connected motor windings, and where  $V_{DC}$  is the continuous inverter input voltage.



**Figure 8: Basic scheme of 3-phase inverter and AC-motor**

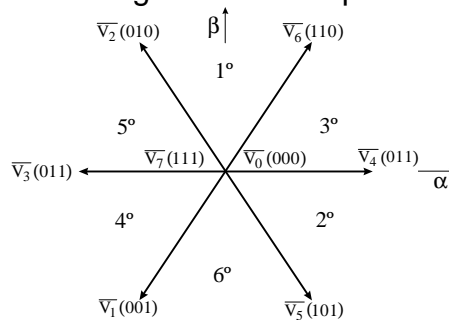
The six switches can be power BJT, GTO, IGBT etc. The ON-OFF sequence of all these devices must respect the following conditions:

- three of the switches must always be ON and three always OFF.
- the upper and the lower switches of the same leg are driven with two complementary pulsed signals. In this way no vertical conduction is possible, providing care is taken to ensure that there is no overlap in the power switch transitions.

The next paragraph presents a technique for generating such pulsed signals.

## 5.2 The Space Vector Pulse Width Modulation (SVPWM)

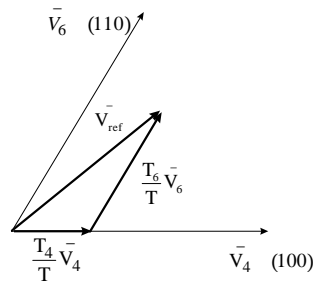
Space Vector PWM supplies the AC machine with the desired phase voltages. The SVPWM [5] method of generating the pulsed signals fits the above requirements and minimizes the harmonic contents. Note that the harmonic contents determine the copper losses of the machine which account for a major portion of the machine losses. Taking into consideration the two constraints quoted above there are eight possible combinations for the switch commands. These eight switch combinations determine eight phase voltage configurations. The diagram below depicts these combinations.



**Figure 9: SVPWM, vectors and sectors**

The vectors divide the plan into six sectors. Depending on the sector that the voltage reference is in, two adjacent vectors are chosen. The binary representations of two adjacent basic vectors differ in only one bit, so that only one of the upper transistors switches when the switching pattern moves from one vector to the adjacent one. The two vectors are time weighted in a sample period  $T$  to produce the desired output voltage.

Assuming that the reference vector  $\vec{V}_{ref}$  is in the  $3^\circ$  sector, we have the following situation:

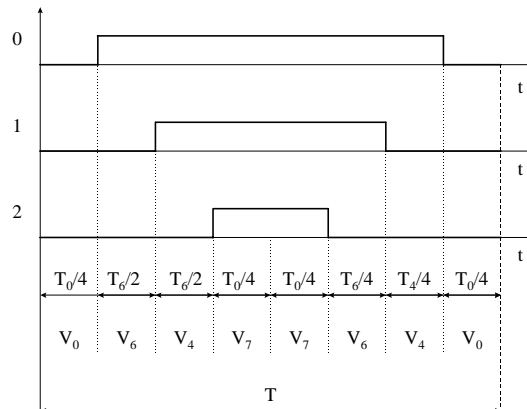


**Figure 10: Reference vector as a combination of adjacent vectors**

Where  $T_4$  and  $T_6$  are the times during which the vectors  $\bar{V}_4$ ,  $\bar{V}_6$  are applied and  $T_0$  the time during which the zero vectors are applied. When the reference voltage (output of the inverse Park transformation) and the sample periods are known, the following system makes it possible to determine the uncertainties  $T_4$ ,  $T_6$  and  $T_0$ .

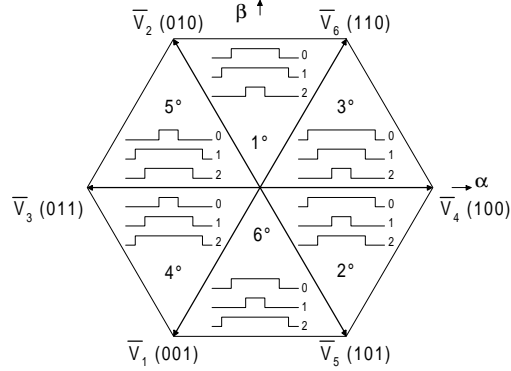
$$\begin{cases} T = T_4 + T_6 + T_0 \\ \bar{V}_{ref} = \frac{T_4}{T} \bar{V}_4 + \frac{T_6}{T} \bar{V}_6 \end{cases}$$

Under these constraints the locus of the reference vector is the inside of a hexagon whose vertices are formed by the tips of the eight vectors. The generated space vector PWM waveforms are symmetrical with respect to the middle of each PWM period [3]. The diagram shows the waveforms in the example presented above.



**Figure 11: Pattern of SVPWM in the sector 3**

The following diagram shows the pattern of SVPWM for each sector:



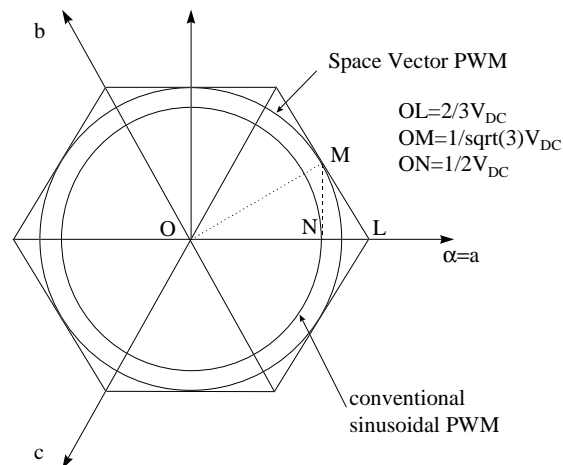
**Figure 12: Hexagon of SVPWM, pattern**

In conclusion, the inputs for the SVPWM are the reference vector components ( $v_{\alpha Sr}$ ,  $v_{\beta Sr}$ ) and the outputs are the times to apply each of the relevant sector limiting vectors.

### 5.3 Comparison SV-sinusoidal PWM

**The SVPWM generates minimum harmonic distortion of the currents in the winding of 3-phase AC motor. SV Modulation also provides a more efficient use of the supply voltage in comparison with sinusoidal modulation methods.** In fact, with conventional sinusoidal modulation [7][8][9] in which the sinusoidal signals are compared with a triangular carrier, we know that the locus of the reference vector is the inside of a circle with a radius of  $\frac{1}{2}V_{DC}$ . In the SV modulation it can be shown that the length of each of the six vectors is  $\frac{2}{3}V_{DC}$ . In steady state the reference vector magnitude might be constant. This fact makes the SV modulation reference vector locus smaller than the hexagon described above. This locus narrows itself to the circle inscribed within the hexagon, thus having a radius of  $\frac{1}{\sqrt{3}}V_{DC}$ . In Figure 13 below the different reference vector loci are presented.





**Figure 13: Locus comparison SV-sinusoidal PWM**

Therefore, the maximum output voltage based on the Space Vector theory is  $\frac{2}{\sqrt{3}} \left( \frac{OM}{ON} \right)$  times as large as that of the conventional sinusoidal modulation. This explains why, with SVPWM, we have a more efficient use of the supply voltage than with the sinusoidal PWM method.

## 6. Conclusion

This paper dealt with the Field Orientated Control of three-phase AC machines. Following a description of common major drawbacks of classic control structures it has been shown how the Field Orientated Control overcomes these deficiencies and what kind of benefits Field Orientated Controlled AC drives can bring. By explaining in detail each of the FOC modules necessary this paper presents a clear introduction to efficient vector control of AC drives. By providing high CPU power and vector control dedicated peripherals in one single TMS320F240 chip, and by giving references to the necessary software modules, Texas Instruments addresses every start-up requirement and allows users of the DSP Controller rapidly to commence development of a system based on vector control with the TMS320F240.

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